Strength of Materials

Through Questions & Answers

For **ESE, GATE, PSUs**

& Other Competitive Examinations

by

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PREFACE

I must thank CMD of MADE EASY Group, **Mr. B. Singh** for providing me an opportunity to reach out to the Student Community at large through my present book **"Strength of Materials through Questions & Answers".** Students may be benefitted from my 50 years of teaching /research experience through this book.

Questions in the book are designed on the pattern of questions that are being asked in university examinations and competitive examinations of UPSC/GATE/PSUs.

The book has been thoroughly and questions from competitive examinations for the last 2 years have been added, in this revised and enlarged edition.

Further improvements in the text book will be made after getting the response from the students.

Any error in printing or calculations pointed out by the reader will be acknowledged with thanks by the author.

Dr. U. C. Jindal
Author

CONTENTS

Simple Stresses and Strains

CHAPTER 1

In this chapter we will discuss various types of external loads that can be applied on a body and the deformation produced in body. Due to the applied load, stresses and strains are developed in a body. Relationship between different types of stresses and different types of strains will be developed.

Positive and negative normal stresses and positive and negative shear stresses will also be described.

Question 1.1 What do you understand by an axial load on a body and axial stress developed in a body.

Solution: Consider a circular bar *AB*, of diameter d and axial length *L* as shown in figure 1.1. End *A* of the bar is fixed and a load *P* is applied along 00 axis of the bar. Axis 00 passes through the centroids of all the sections of the bar, as shown. Load *P* is perpendicular to all sections of the bar. The load applied can be a point load along the axis 00 or a uniformly distributed load over the section of the bar. To maintain equilibrium, a reaction *R* = *P* is developed at the fixed end.

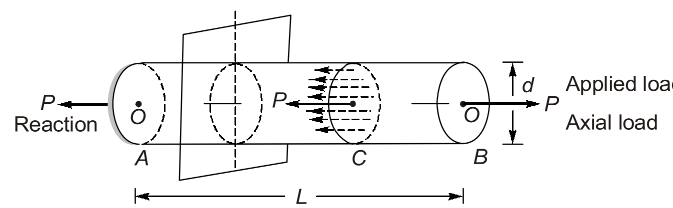


Fig. 1.1

If you consider a portion BC of the bar *AB*, then a force *P* at section *B* is resisted by equal and opposite force at section *C*, this equal and opposite force *on internal section* is known as internal resistance. This internal resistance *P* per unit area is defined as stress.

$$\text{or stress,} \qquad \sigma = \frac{P}{\text{area of section}} = \frac{4P}{\pi d^2}$$

Note that force *P* is perpendicular to section.

Therefore stress σ is normal to the section.

Stress σ is perpendicular to the section of the bar, and is defined as normal stress. Direct stress is along the axis of bar. If axial load *P* is expressed in N (newton) and area of cross-section in mm^2, then units of stress are N/mm^2 (newton per mm^2). Stress is a second order vector, it has both magnitude and direction.

Moreover load is normal to the section and its direction is away from the plane. Note that load *P* is normal to plane *and is pointing away from the plane*.

Similarly internal resistance *P* is normal to plate at *C* and is pointing away from the plane. This type of load is known as tensile load and stress produced by tensile load is tensile stress (or positive normal stress).

Question 1.2 Explain what is tangential load or shear load and what are shear stress and shear strain?

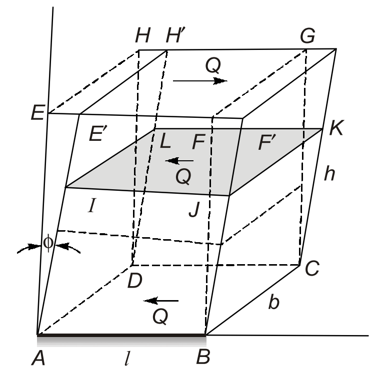


Fig. 1.2

Solution: Consider a rectangular block (parallelopiped block) of dimensions *l*, *b*, *h* as shown in figure. 1.2.

Lower face of block, *ABCD* is fixed and a force *Q* parallel to the upper face *EFGH* is applied.

To maintain equilibrium, reaction *Q* (equal and opposite to *Q* on top face) is developed or consider an internal plane IJKL, parallel to top face. Internal resisting force *Q*, parallel to the internal plane is developed. **This internal resistance per unit area is termed as shear stress, τ**

$$\tau \text{, shear stress} = \frac{\text{Tangential force } Q}{\text{Area } l \times b} = \frac{Q}{l.b} \text{ in N/mm}^2$$

If Q is in Newton (N) and area in mm^2.

This ***tangential load on a surface is also known as shear load.*** Due to this shear load, block is deformed i.e.., rectangle *ABFE* is changed to parallelogram ABF′E′.

$$\angle EAE' = \phi \text{, shear angle}$$

$$\phi = \tan^{-1}\frac{EE'}{EA} = \tan^{-1}\frac{EE'}{h}$$

This shear angle ϕ is very small, so $\tan\phi \approx \sin\phi \approx \phi$. That is also known as *shear strain*.

Question 1.3 Explain what is Hooke's law?

Solution: Hooke's law states that stress (σ) is proportional to strain (ε)

$$\sigma \propto \varepsilon$$

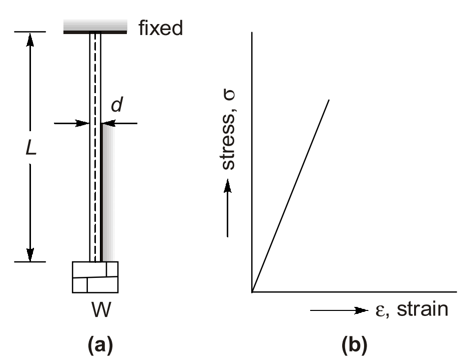


Fig. 1.3

Take a wire of diameter *d*, length *L*, fixed at upper end and a load *W* is gradually applied at the lower end.

Due to the tensile load the wire gets extended say by an amount δL.

Tensile stress in wire, $\sigma = \dfrac{4W}{\pi d^2}$

Change in length per unit length,

$$\varepsilon = \frac{\delta L}{L} \text{ or } \delta L = \varepsilon L.$$

Change in ***length per unit length is termed as strain, ε***.

If a graph is plotted between stress (σ) and strain (ε), for a gradually increasing load *W*, then within the elastic limit.

$$\sigma \propto \varepsilon$$

Elastic limit, means that within the elastic limit on stress i.e., σ_e, if load is removed from the wire, then **residual strain (residual change in length) will be zero.**

$$\sigma = E\varepsilon$$

where E = Constant of proportionality

= Elastic constant

$$= \text{Young's modulus of elasticity}$$
$$= \sigma/\varepsilon$$

Similarly for the shear load on rectangular block, shear stress α shear strain.

or shear stress, $\tau \propto \phi$, shear strain

$$\tau = G\phi$$

which G is the constant of proportionality

$$G = \frac{\tau}{\phi} = \frac{\text{shear stress}}{\text{shear strain}} = \text{Shear modulus}$$

E and G are elastic constants and are given for the material by the manufacturer.

Question 1.4 A circular stepped bar made of steel is shown in figure 1.4.

Lengths AB = 40 mm, BC = 60 mm, CD = 80 mm.

Diameters, d_1 = 20 mm, d_2 = 15 mm, d_3 = 10 mm.

An axial tensile load P = 10 kN is applied on the bar as shown.

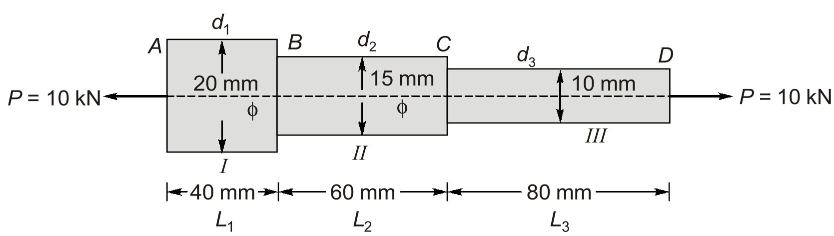


Fig. 1.4

Determine stresses, strains and change in length in each portion. For steel E = 200 GPa = 200 kN/mm^2.

Solution: Axial tensile load is perpendicular to all sections of portions AB, BC and CD. Stress is load per unit area, so maximum stress will occur in portion III **with minimum diameter**. Similarly minimum stress will occur in portion-I of maximum diameter.

Diameters, d_1 = 20 mm, d_2 = 15 mm and d_3 = 10 mm

Areas of cross-section, $A_1 = \frac{\pi}{4} \times 20^2 = 314.16 \text{ mm}^2$

$$A_2 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ mm}^2$$

$$A_3 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ mm}^2$$

Force, P = 10,000 N

Stresses, $\sigma_1 = \frac{10000}{314.16} = 31.83 \text{ N/mm}^2$

$$\sigma_2 = \frac{10000}{176.7} = 56.59 \text{ N/mm}^2$$

$$\sigma_3 = \frac{10000}{78.54} = 127.32 \text{ N/mm}^2$$

All these stresses are tensile stresses.

Strains $E = 200{,}000\ \text{N/mm}^2$

Using Hooke's law

$$\varepsilon_1 = \frac{\sigma_1}{E} = \frac{31.83}{200000} = 1.59\times10^{-4} \text{ tensile strain}$$

$$\varepsilon_2 = \frac{\sigma_2}{E} = \frac{56.59}{200000} = 2.83\times10^{-4} \text{ tensile strain}$$

$$\varepsilon_3 = \frac{\sigma_3}{E} = \frac{127.32}{200000} = 6.36\times10^{-4} \text{ tensile strain}$$

Chang in length, $\delta L_1 = \varepsilon_1 \times L_1 = 1.59 \times 10^{-4} \times 40 = 0.00636$ mm, extension

$\delta L_2 = \varepsilon_2 \times L_2 = 2.83 \times 10^{-4} \times 60 = 0.01698$ mm, extension

$\delta L_3 = \varepsilon_3 \times L_3 = 6.36 \times 10^{-4} \times 80 = 0.0508$ mm, extension

Practice Q.1.4 A circular stepped bar made of copper is shown in figure 1.5. It is subjected to axial load *P* such that maximum stress in bar is not to exceed 100 MPa. What is the magnitude of *P*? What are strains in two portions. What is the total change in length, *E* for copper = 105 GPa = 105 kN/mm^2.

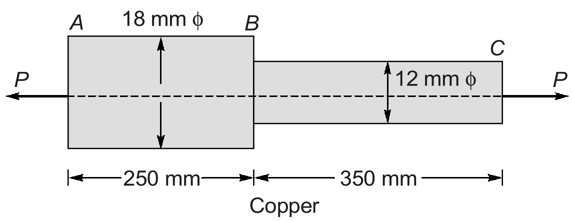


Fig. 1.5

Ans. [11.31 kN, 4.233×10^{-4}, 9.524×10^{-4}, 0.439 mm]

Question 1.5 Explain (a) Tensile and compressive stresses (b) Positive and Negative shear stresses.

Solution: Tensile and compressive stresses are normal stresses, i.e., their direction is perpendicular to the plane.

Direction of a tensile stress is away from the plane, while direction of compressive stress is towards the plane.

Shear stress is tangential to the plane, positive shear stress tends to rotate the body in a clockwise direction, while negative shear stress tends to rotate the body in anticlockwise direction.

Consider a bar of rectangular section $b \times h$ subjected to axial tensile load P as shown in figure 1.6. Take an inclined plane *abcd*, with side *ad* inclined to axes at an angle α.

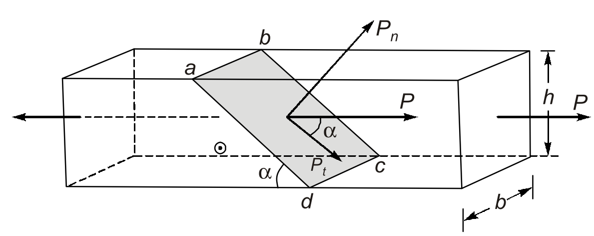


Fig. 1.6

Section *abcd* is an internal section and P is the internal resistance of the body. Force P has two components

$P \sin\alpha = P_n$, normal component

$P \cos\alpha = P_t$, tangential component

P_n is pointing away from the plane. It is a **positive normal force** or tensile force.

$$\sigma = \text{normal stress in plane} = \frac{P_n}{\text{area}} = \frac{P_n}{ad \times dc}; \text{ side } ad = \frac{h}{\sin\alpha}$$

$$= \frac{P_n}{\frac{h}{\sin\alpha} \times b} = \frac{P_n \sin\alpha}{hb}, \text{ putting the value of normal force}$$

$$= \frac{P\sin\alpha.\sin\alpha}{hb} = \frac{P\sin^2\alpha}{hb}$$

P_t is a force tangential to the plane *abcd*, if we take moment of the force **about point ⊙ on body**, the force P_t tends to rotate the body in clockwise direction.

This is a positive shear force on the plane

Positive shear stress, $$\tau = \frac{P\cos\alpha}{ad \times dc} = \frac{P\cos\alpha}{h \times b} \times \sin\alpha$$

$$= \frac{P\sin\alpha\cos\alpha}{bh}; \text{ tends to rotates the body in clockwise direction.}$$

Now consider a rectangular bar of section $b \times h$, subjected to axial compressive load P as shown in figure 1.7.

Take inclined plane *abcd*, side *ad* inclined at an angle α with the axis of the bar

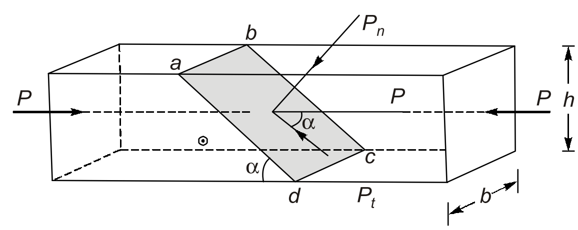


Fig. 1.7

$$\text{side } ad = \frac{h}{\sin\alpha}$$

$$\text{side } ab = b, \text{ breadth}$$

$$\text{area of cross-section} = \frac{h}{\sin\alpha} \times b = \frac{bh}{\sin\alpha}$$

Normal compressive stress on plane

$$\sigma = \frac{P_n}{bh}\sin\alpha$$

where, $$P_n = P\sin\alpha$$

so $$\sigma = \frac{P\sin^2\alpha}{bh},$$ Note that P_n is **pointing towards the plane**

It is a **compressive force** (a negative normal force), σ is a compressive stress.

Consider the effect of tangential force P_t about a point ⊙ on the body, it tries to rotate the body in anticlockwise direction. It is a negative shear force

Shear stress, $$\tau = \frac{P_t}{\text{area}} = \frac{P\cos\alpha}{\frac{bh}{\sin\alpha}} = \frac{P\sin\alpha\cos\alpha}{bh}$$

So a negative **shear stress tends to rotate the body in an anticlockwise direction.**

Question 1.6 Consider figure 1.7, with b = 20 mm, h = 30 mm, and P (compressive force) = 10 kN. Inclined plane at a angle α = 60° with the axis of the bar. Determine normal and shear stresses an inclined plane.

Solution:

$$P = 10 \text{ kNm}, \quad \alpha = 60°$$

$$P_n = P \sin\alpha = 10 \times \sin 60° = 5\sqrt{3} \text{ kN} = 8660.25 \text{ N}$$

Area of cross-section of inclined of plane

$$= \frac{b \times h}{\sin\alpha} = \frac{20 \times 30}{\sin 60°} = 692.82 \text{ mm}^2$$

$$\text{Normal stress on plane} = -\frac{P_n}{692.82} = -\frac{8660.25}{692.82} = -12.5 \text{ N/mm}^2$$

$$\text{Shear stress as plane (negative), } \tau = \frac{P_t}{bh}\sin\alpha$$

$$P_t = P\cos\alpha = P\cos 60°$$

$$= 10000 \times 0.5 = 5000 \text{ N}$$

$$\text{Shear stress (negative), } \quad \tau = \frac{P_t}{\text{area}} = \frac{5000}{692.82} = 7.216 \text{ N/mm}^2$$

(tends to rotate the body in anticlockwise direction)

Practice Q.1.6 A circular bar of diameter 20 mm in subjected to an axial tensile force of 15 kN. Determine normal and shear stress on an inclined plane. Angle of inclination of the plane with axes of the bar is 30° (α).

Hints:

$$\text{Area of an ellipse} = \pi \times \text{semi minor axis} \times \text{semi major axis}$$

$$\text{Semi major axis} = \frac{1}{2} \times \frac{d}{\cos\alpha} = \frac{d}{1.732}$$

$$\text{Semi minor axis} = \frac{1}{2}A = 0.5d$$

Ans. [35.81 N/mm², 20.67 N/mm²]

Question 1.7 What do you understand by complementary shear stresses, explain with the help of a neat sketch.

Solution:

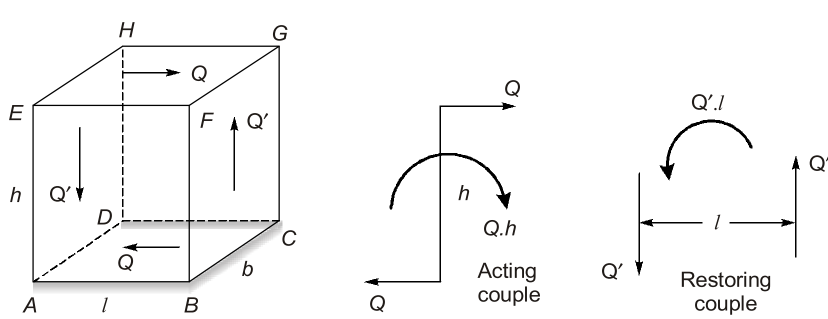


Fig.1.8

Consider a rectangular block of dimensions $l \times b \times h$, fixed at bottom, subjected to a shear force Q on top surface as shown in figure 1.8.

A reaction force Q acts at bottom surface.

There two forces forming a couple of arm h.

Couple moment $M = Q.h$ (clockwise)

Under the action of couple $Q.h$ the block will try to rotate. To maintain equilibrium an equal, and opposite couple will act on the block. Say forces $Q'\uparrow$ and $Q'\downarrow$ act on vertical faces as shown.

Arm of the couple is l.

$Q'.l$ ((anticlockwise couple) reaction couple

$$Q.h = Q'l$$

but $Q = \tau \cdot l \cdot b$, where τ is shear stress on horizontal top and bottom faces

$$\tau.l.b.h = Q'l$$

$$Q' = \tau.b.h = \tau' \times bh$$

where τ' is shear stress on vertical faces, perpendicular to horizontal faces.

$$\tau = \tau'$$

τ' is complementary shear stress to the applied shear stress τ. Note that both these stresses are perpendicular to each other.

Similarly shear stress τ is complementary to shear stress τ'.

Definition: Whenever a shear stress is applied on a certain plane, a complementary shear stress of same magnitude but opposite in nature on a perpendicular plane is developed to maintain equilibrium.

Further note that shear stress τ on horizontal surfaces tends to rotate the body in clockwise direction. Whereas τ', shear stress on vertical faces tends to rotate the body in anticlockwise direction so as to maintain equilibrium.

Question 1.8 **Explain longitudinal strain, lateral strain and Poisson's ratio.**

Solution: When a bar is subjected to an axial tensile force, its length is increased but its diameter is decreased. The change in length per unit length is termed as *longitudinal strain*.

The change in diameter per unit diameter is termed *as lateral strain. The ratio of lateral strain/longitudinal strain is called Poisson's ratio.* Similarly when bar is subjected to axial compressive load, its length is decreased but its diameter is increased. The ratio of change in length per unit length is longitudinal strain (negative) change in diameter (increase) per unit diameter is known as lateral strain (Positive strain). Diameter is in lateral direction to axis of the bar.

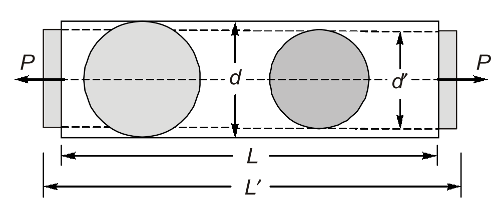


Fig. 1.9

Consider a bar of length L, diameter d subjected to axial tensile load, P. Its length is increased to L', such that $L' > L$, its diameter is decreased, $d' < d$.

Change in length $\delta L = L' - L$

longitudinal strain, $\varepsilon = \dfrac{\delta L}{L} = \dfrac{(L' - L)}{L}$, positive strain

change in diameter, $\delta d = d' - d$

Lateral strain, $\varepsilon' = \frac{\delta d}{d} = \left(\frac{d' - d}{d}\right)$, negative strain

Poisson's ratio, $\nu = \frac{\varepsilon'}{\varepsilon} = -\frac{\delta d}{d} \times \frac{L}{\delta L}$ = a negative ratio.

But generally the negative sign is not attached with the Poisson's ratio.

Lateral strain = $-\nu \times$ longitudinal strain

Using Hooke's law

Axial tensile load = P

Original area of cross-section = $\frac{\pi}{4} d^2$

Axial stress, $\sigma = \frac{4P}{\pi d^2}$

Longitudinal strain, $\varepsilon = \frac{\sigma}{E} = \frac{\text{Axial stress}}{\text{Young's modulus}}$

Lateral strain, $\varepsilon' = -\nu\varepsilon = -\nu\frac{\sigma}{E}$

Question 1.9 A steel round bar of diameter 16 mm, length 250 mm is subjected to an axial compressive load of 16 kN. (fig. 1.9). If E = 200 GPa, Poisson's ratio = 0.3 for steel, determine (a) axial stress (b) axial or longitudinal strain (c) lateral strain (d) change in length (e) change in diameter. For steel, E = 200 GPa, ν = 0.3 (Poisson's ratio).

Solution:

Original diameter, d = 16 mm

Axial compressive load, P = 16 kN

Length, L = 250 mm

Area of cross-section, $A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 16^2 = 201.06 \text{ mm}^2$

Axial stress, $\sigma = -\frac{P}{A} = -\frac{16000}{201.06} = -79.57 \text{ N/mm}^2$ (compressive stress)

Axial or longitudinal strain, $\varepsilon = -\frac{\sigma}{E} = -\frac{79.57}{200000} = -0.398 \times 10^{-3}$

Change in length, $\delta L = -\varepsilon L = -0.398 \times 10^{-3} \times 250 = -0.0995$ mm

Lateral strain,, $\varepsilon' = -\nu\varepsilon = +0.3 \times 0.398 \times 10^{-3} = +0.1194 \times 10^{-3}$

Change in diameter, $\delta d = +0.1194 \times 10^{-3} \times 16 = 1.91 \times 10^{-3}$ mm

Practice Q.1.9 A bar of rectangular section 20 × 30 mm is subjected to axial tensile load of 30 kN. Length of bar is 300 mm. Bar is made of copper. For copper E = 105000 N/mm^2 and Poisson's ratio, ν = 0.35. Determine (a) axial stress (b) axial strain (c) lateral strain (d) change in length (e) changes in breadth and thickness.

Ans. [+50 N/mm^2, + 0.476 × 10^{-3}, –0.1667 × 10^{-3}, + 0.1428 mm, –3.33 × 10^{-3} mm, –5 × 10^{-3} mm]

Question 1.10 A rectangular block of copper with base 40 × 60 mm and height 80 mm is fixed in the ground. It is subjected to a shear force on top face equal to 24 kN. What are shear stress, shear strain developed in block if G for copper 39 kN/mm^2. What is displacement AA'?

Solution:

Shear force, $Q = 24$ kN

Area of top face $= 60 \times 40 = 2400$ mm^2

Shear stress, $\tau = \dfrac{24000}{2400} = 10\text{N/mm}^2$

(it tries to rotate the block in clockwise direction about point ⊙)

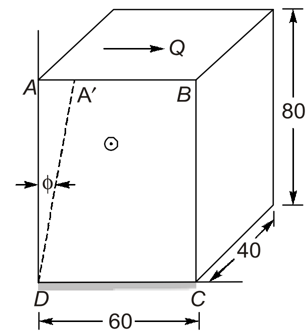


Fig. 1.10

Shear strain, $\phi = \dfrac{\tau}{G} = \dfrac{10}{39000} = 0.2564 \times 10^{-3}$ radian

Displacement, $AA' = \phi h = 0.2564 \times 10^{-3} \times 80 = 0.0205$ mm

Question 1.11 Explain what is stress tensor?

Solution: Stress tensor is a representation of normal and shear stresses on a three dimensional block of size $\Delta x = \Delta y = \Delta z \to 0$. In other words it represents state of stresses at a point along three orthogonal directions.

σ stands for normal stress

τ stands for shear stress

σ_{xx}, Ist subscript x denotes direction of normal to the plane (yz plane), x second subscript denotes direction of normal stress.

τ_{xy}, Ist subscript x denote direction of normal to the plane (yz plane) and y second subscript denotes direction of shear stress in y direction. Similarly

τ_{yz}, Ist subscript y denotes direction of normal to the plane (xz plane) and second subscript z denote direction of shear stress in z-direction.

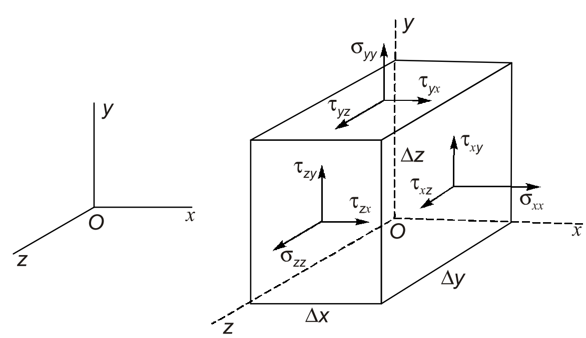


Fig. 1.11 *Normal and shear stresses on 3 orthogonal planes*

Stress tensor is represented in matrix form as follows:

$$\tau_{ij} = \begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{vmatrix}$$

Stresses along diagonal, i.e., σ_{xx}, σ_{yy}, σ_{zz} are normal stresses.

In a two dimensional case, stress tensor,

$$\tau_{ij} = \begin{vmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{vmatrix}$$

Shear stress $\tau_{xy} = \tau_{yx}$, $\tau_{zx} = \tau_{xz}$, $\tau_{zy} = \tau_{yz}$ are complementary shear stresses.

Question 1.12 Consider a uniformly tapered circular bar, of axial length L, diameters d and D, subjected to axial load. E is the modulus of elasticity of the material of the bar. Derive expression for change in its axial length.

Solution: Fig. 1.12 shows a circular tapered bar. Take a strip of diameter d_x at a distance of x from end A.

Diameter, $d_x = d + \frac{D-d}{L}x = d + kx$ where k = $\frac{D-d}{L}$ a ratio

Area of cross-section $= \frac{\pi(d+kx)^2}{4}$

Axial load $= P$

σ_x, stress at elementary strip $= \frac{4P}{\pi(d+kx)^2}$

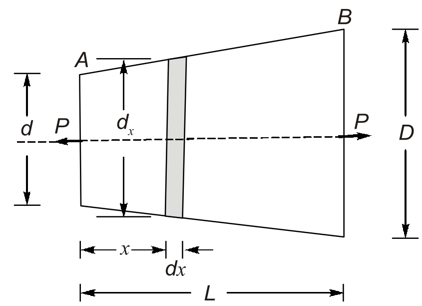


Fig. 1.12

Strain at elementary strip, $\varepsilon_x = \frac{4P}{E\pi(d+kx)^2}$

Change in length over length dx,

$$\delta dx = \frac{4Pdx}{E\pi(d+kx)^2}$$

Total change in length, $\delta L = \int_0^L \frac{4Pdx}{E\pi(d+kx)^2}$

$$= -\left|\frac{4Pdx}{E\pi(d+kx)}\right|_0^L = -\frac{4P}{\pi Ek}\left|\frac{1}{d+kL} - \frac{1}{d}\right|$$

Putting the value of k,

Change in length, $\delta L = -\frac{4PL}{\pi E(D-d)}\left[\frac{1}{D} - \frac{1}{d}\right]$

$$= -\frac{4PL}{\pi E(D-d)}\left[\frac{d-D}{dD}\right] = +\frac{4PL}{E\pi Dd}$$

Practice Q.1.12 An aluminium circular bar tapers uniformly from diameter 20 mm to diameter 30 mm over length L = 500 mm. If an axial tensile load of 40 kN in applied on bar. What is the change in length of the bar, if E = 67 GPa.

Ans. [0.633 mm]

Question 1.13 What do you mean by volumetric stress, volumetric strain and bulk modulus of elasticity.

Solution: Volumetric stress is a stress which acts with same magnitude in all directions such as hydrostatic pressure, p. As per Pascal's law, liquid exerts equal pressure in all directions.

As an example, if a ball is immersed in sea water, pressure on ball, $p = wh$, where w is weight density of sea water and h is depth of the ball in sea water.

If the depth of immersion is increased gradually then pressure on ball also increases simultaneously. Diameter of the ball gradually is reduced producing diametral strain in bar and consequently, the volumetric strain, which is three times the diametral strain. It a graph is plotted between pressure, and volumetric strain, ε_v an shown in figure 1.13, then upto proportional limit A, pressure is directly proportional to volumetric strain.

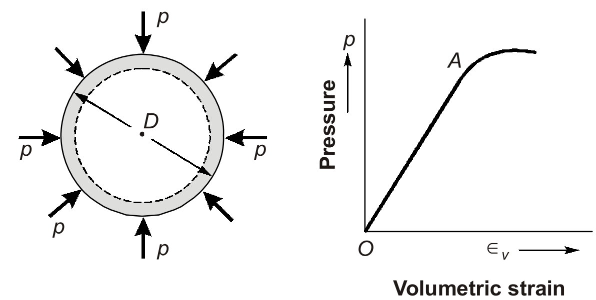


Fig. 1.13

Pressure, $p \propto e_v$, volumetric strain

or $p = Ke_v$

K is proportionality constant, called Bulk modulus

Bulk modulus, $K = \dfrac{\text{pressure, } p}{\text{volumetirc strain, } \varepsilon_v}$

Question 1.14 A spherical ball of steel of diameter 200 mm goes down to a depth of 1 km in sea water. If specific weight of sea water in 10.2 kN/m^3, (and bulk modulus of steel is 170 kN/mm^2), determine change in volume of spherical ball.

Solution:

Depth, $h = 1000$ m

Weight density, $w = 10.2 \times 1000$ N/m^3

Pressure, $p = wh = 1000 \times 10.2 \times 1000$ N/m^2

$= 10.2 \times 10^6$ N/m^2

Bulk modulus, $K = 170 \times 10^3$ N/mm^2 $= 170 \times 10^9$ N/m^2

Volumetric strain, $\varepsilon_v = \dfrac{p}{K} = \dfrac{10.2\times10^6}{170\times10^9} = 0.06\times10^{-3}$

Original volume, $V = \dfrac{\pi D^3}{6} = \dfrac{\pi\times200^3}{6} = 4.188\times10^6\,\text{mm}^3$

Change volume, $\delta V = \varepsilon_V.\ V = 0.06 \times 10^{-3} \times 4.188 \times 10^6$

$= 0.25 \times 10^3$ mm^3

$= 250$ mm^3 $= 0.25$ cc

Practice Q.1.14 A spherical ball of copper of diameter 300 mm is immersed in sea water specific weight of sea water is 10.2 kN/m^3. Bulk modulus of copper is 98 GPa. For how much depth ball must be immersed that change in volume is 10 cc.

Ans. [6.796 km]

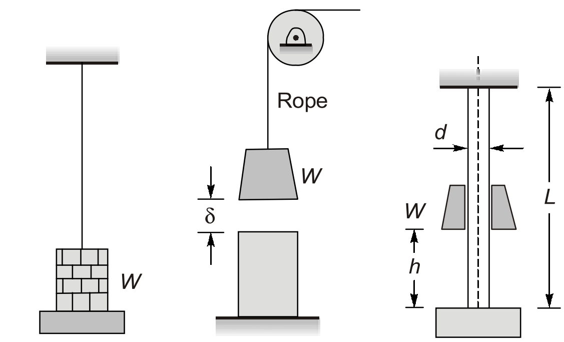
Question 1.15 Differentiate between gradual, sudden and shock loads. Derive expressions of stresses due to sudden load and due to shock load.

Solution:

Gradual Load: Gradual load is continuously increasing load starting from zero. As the load is gradually increased, stress σ and strain ε in the body also gradually increase. Fig. 1.14 (a) shows a wire subjected to gradually increasing load W on hanger.

$$\sigma_{gradual} = \frac{\text{Load}}{\text{Area}} = \frac{W}{\text{Area}}$$

Sudden Load: Load is not gradually increased but **whole of the magnitude of the load acts at once or suddenly** on the body. Fig. 1.14 (b) shows a load W being lowered slowly through a wire rope of a crane. When the gap , δ between body and load tends to become zero, the wire rope breaks and **whole of the load W acts suddenly on the column** (as shown). Say the change in length in body δL.



(a) Gradual load (b) Sudden load (c) Shock load

Fig. 1.14

Work done on body by the sudden load = $W.\delta L$

Internal resistance of the body does not develop suddenly but it takes time and develops gradually **as shown by resistance R in figure 1.15.**

Stress is defined as internal resistance per unit area.

Workdone on the body is absorbed by the body as strain energy.

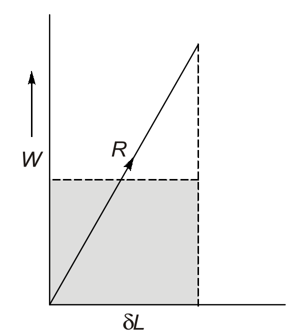


Fig.1.15 *Sudden load*

$$\text{Internal energy} = \frac{1}{2}R.\delta L = W.\delta L$$

(where R denotes internal resistance)

$$\text{Internal resistance,} \quad R = 2W$$

$$\text{Stress due to sudden load,} \quad \sigma_{sudden} = \frac{R}{\text{area}} = \frac{2W}{A} = 2 \times \sigma_{gradual}$$

$\frac{W}{A}$ is the gradual stress as the load W is applied gradually starting from zero.

When the load is applied through some velocity or the load possesses kinetic energy, as the small bullet coming out of the muzzle of a gun with very high velocity produces tremendous stress depending upon the energy.

Figure 1.14 (c) shows a bar of length L, diameter d fixed at upper end and carries a collar at the lower end. A load W is allowed to fall freely on the collar through height 'h'. When the load falls under gravity, velocity $V = \sqrt{2gh}$ is gained by weight, W. Load W is arrested by the collar. Say due to shock load, δL_i is instantaneous extension in bar and say σ_i is the instantaneous stress developed in the bar.

$$\text{Loss of potential energy of weight} = W(h + \delta L_i)$$

$$\text{Gain in strain energy of the bar} = \sigma_i A(\delta L_i)\frac{1}{2}$$

and

$$W(h + \delta L_i) = \frac{\sigma_i A}{2}\delta L_i \quad \text{...(i)}$$

Using Hooke's law

$$\delta L_i = L \times \varepsilon_i' = L \times \text{instantaneous strain}$$

$$= L \times \frac{\sigma_i}{E}$$

Putting these values in equation (i)

$$W\left(h + L\frac{\sigma_i}{E}\right) = \sigma_1 \frac{A}{2} \times L \times \frac{\sigma_i}{E}$$

or
$$Wh + WL\frac{\sigma_i}{E} = \frac{\sigma_i^2}{2E} \times AL$$

or
$$\frac{Wh \times 2E}{AL} + \frac{\sigma_i}{E} \times WL \times \frac{2E}{AL} = \sigma_i^2$$

$$\sigma_i^2 - \frac{2EWh}{AL} - \sigma_i \frac{2W}{A} = 0$$

A quadratic in σ_i, solution of this equation

$$\sigma_i = \frac{1}{2}\left[\frac{2W}{A} \pm \sqrt{\frac{4W^2}{A^2} + \frac{8WEh}{AL}}\right] = \frac{W}{A} \pm \frac{W}{A}\sqrt{1 + \frac{2EAh}{WL}}$$

Negative sign is inadmissible, because the type of the load shown does not produce compressive stress.

$$\sigma_i = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2EAh}{WL}}\right] \qquad \text{...(ii)}$$

If you analyse this expression, then σ_i' depends on the magnitude of W and height h.

If height $h = 0$, then $\sigma_i = \sigma_{sudden} = \frac{2W}{A}$

Many a times, loss of *PE* or gain in *KE* is so large that stress produced far exceeds σ_{sudden}.

Question 1.16 Consider a steel bar of diameter 10 mm, length 500 mm, fixed at upper end and carries a collar at the lower end. A load *W* of 1 kN falls through height *h*. Determine *h* if the instantaneous stress developed in bar is 150 N/mm². *E* = 208000 N/mm².

Solution:

$$\sigma_i = 150 \text{ N/mm}^2$$
$$W = 1000 \text{ N}$$
$$A = \frac{\pi}{4} \times 10^2 = 78.54 \text{ mm}^2$$
$$L = 500 \text{ mm}$$
$$\frac{W}{A} = \frac{1000}{78.54} = 12.73$$
$$E = 208000 \text{ N/mm}^2$$

Putting the values in the expression for instantaneous stress

$$150 = 12.73\left[1 + \sqrt{1 + \frac{2 \times 208000 \times h}{12.73 \times 500}}\right]$$

$$\left(\frac{150}{12.73}-1\right)^2 = 1 + 65.35\,h$$

$$116.277 = 1 + 65.35\,h$$

Height, $$h = \frac{116.277-1}{65.35} = 1.764 \text{ mm}$$

If the load $W = 1$ kN, which can only produce gradual stress of 12.73 N/mm^2, is allowed to fall through a height of 1.764 mm only then instantaneous stress produced in the bar is 150 N/mm^2.

Practice Q.1.16 A copper bar of diameter 15 mm, length 600 mm in fixed at top end and at bottom end a collar is provided. A load of 500 N is allowed to fall through a height of 30 mm on the collar. If E for copper is 105000 N/mm^2, what is the instantaneous stress developed in the bar.

Ans. [175.2 N/mm^2]

Question 1.17 What do you understand by strain energy absorbed by a body subjected to external loads? Differentiate between resilience, proof resilience, modulus of resilience and toughness.

Solution: Figure 1.16 (a) shows a bar of length L, area of cross-section A, subjected to axial load P, if E is the modulus of elasticity of the material, then,

stress, $$\sigma = \frac{P}{A}$$

Strain, $$\varepsilon = \frac{\sigma}{E} = \frac{P}{AE}$$

Change in length, $$\delta L = \varepsilon L = \frac{PL}{AE}$$

Work done on bar $$= \frac{1}{2}P\delta L$$

= Strain energy absorbed by bar due to strain/deformation developed in bar

Strain energy, $$U = \frac{1}{2}P\delta L = \frac{1}{2}P \times \frac{PL}{AE} = \frac{P^2L}{2AE}$$

$$= \frac{P^2L}{2A^2E} \times A = \frac{1}{2}\frac{P^2}{A^2} \times AL$$

$$= \frac{1}{2}\frac{\sigma^2}{E}AL = \frac{1}{2}\frac{\sigma^2}{E} \times \text{Volume of bar}$$

Strain energy per unit volume

$$= \frac{\sigma^2}{2E}$$

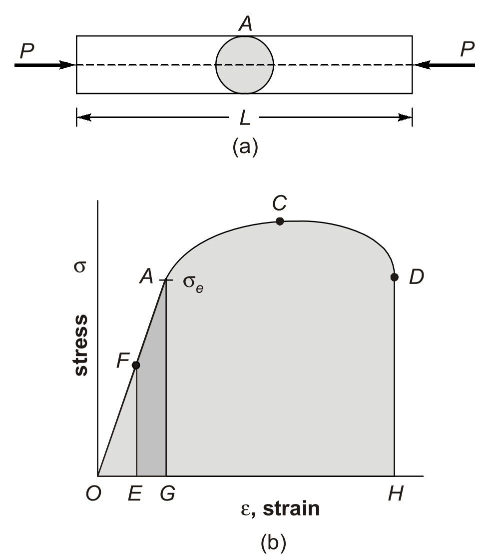


Fig. 1.16

If a graph is plotted between stress σ and strain, ε upto the destruction of the bar, as the shown in figure 1.16 (b) for a ductile material, then

A = limit of proportionality or elastic limit

If load is removed from the bar upto the elastic limit, there is no residual strain remaining in the bar after the removal of the load. Fig. 1.17 shows removal of the load from the bar in the plastic stage.

Point A: limit of proportionality

Point B: elastic limit

Point B is above point A and very close to point A.

If load is removed from point *B* i.e., elastic stage, there will not be any residual strain in bar. If load is removed from the bar in the plastic stage, the unloading curve will be parallel to line *OA*, and there will be residual strain, ε_R remaining in the bar.

Refer to the figure 1.16 (b)

Resilience is the total strain energy absorbed by the body within the elastic limit. Area *OFE* in figure. 1.16(b) shows resilience of the body. If load is removed from point *F*, total energy absorbed by bar is fully recovered.

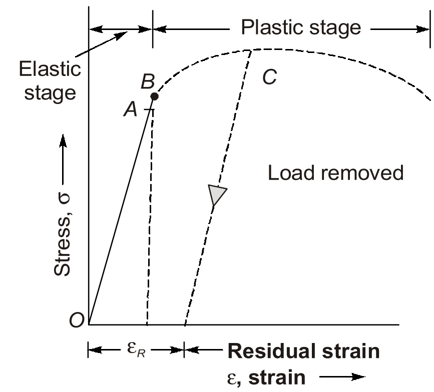


Fig. 1.17

Proof resilience is the total strain energy absorbed by a body upto the elastic limit, i.e., $\dfrac{\sigma_e^2}{2E}\times$ Volume

Proof resilience is the strain energy absorbed by a body per unit volume upto the elastic limit.

$$\text{Proof resilience} = \frac{\sigma_e^2}{2E}, \text{ where } \sigma_e \text{ is the stress at elastic limit in the body.}$$

Toughness is given by the total energy absorbed by a body upto the stage of breaking i.e., area where the curve *OACDH* fig. 1.16(b). In place of a graph between $\sigma - \varepsilon$, if graph is plotted between load, *P* and extension δL, then total energy given under the curve gives the value of toughness of a material.

Toughness is the most desired mechanical property of a material used in engineering components as crank shaft, connecting rod etc. This property is a combination of strength and ductility of a body.

Question 1.18 A cube of side 50 mm in subjected to shear stress and a graph between shear stress and shear strain is plotted, it is observed that elastic limit shear stress is τ_e = 230 N/mm^2. If *G* = shear modulus of material is 83 kN/mm^2, determine

(a) proof resilience (b) modulus of resilience (c) resilience at τ = 120 N/mm^2

Solution:

$$\text{Volume of cube} = 50^3 = 125000 \text{ mm}^3$$

(a) $$\text{Proof resilience} = \text{Total strain energy upto elastic limit}$$

$$= \frac{\tau_e^2}{2G}\times\text{Volume} = \frac{230^2}{2\times 84000}\times 125\times 10^3$$

$$= 39360 \text{ Nmm} = 39.36 \text{ Nm}$$

(b) $$\text{Modulus of resilience} = \frac{\tau_e^2}{2G} = \frac{230^2}{2\times 84000} = 0.314 \text{ Nmm/mm}^3$$

(c) $$\text{Resilience at } \tau = 120 \text{ N/mm}^2 = \frac{120^2}{2G}\times \text{Volume} = \frac{120^2}{2\times 84000}\times 125\times 10^3$$

$$= 10714 \text{ Nmm} = 10.714 \text{ Nm}$$